

# Year 12 Mathematics: Specialist

## Mathematical Investigation

### Topic 3.3 – Vectors in Three Dimensions



### The Problem with Mosquitoes

In this investigation you will use vectors to represent the position and velocity of two mosquitoes in space.

Consider two mosquitoes, A and B, set up the following:

- Mosquito A has initial position  $(a, b, c)$  and has velocity vector  $[d, e, f]$ .
- Mosquito B has initial position  $(g, h, l)$  and has velocity vector  $[m, n, p]$ .

In this investigation, using varying initial positions and velocity vectors for your mosquitoes, you investigate the location and speed of the mosquitoes and the distance they are apart.

In addition, you need to determine the best position for the spray nozzle of an insect repellent to enable you to hit both mosquitoes with one spray.

#### PART 1

For some (sensibly) chosen values of  $a, \dots, h, l, m, n, p$  find the parametric equations for the position of each mosquito at time  $t$ , where  $t$  is the time in seconds, using *centimetres* for the distance.

What is the speed of each mosquito and how long does it take each mosquito to reach a person (or two people) at a point of your choice along its straight line path.

#### PART 2

If at time  $t$ , mosquito A is at position P and mosquito B is at position Q, determine the vector  $\vec{PQ}$  and find their distance apart at any instant. Find the time when the mosquitoes are closest to each other and their distance apart at this instant.

#### PART 3

Now investigate the best location for you to be holding the insect repellent to hit both mosquitoes with just one spray. The sprayed repellent may be represented by a line in 3D. Designate a position and velocity of sprayed insect repellent so that both mosquitoes will be within its range.

Modify your model if possible to consider the insect spray as a plane in 3D or a 3D shape.

#### PART 4

Analyse and interpret your results, including consideration of the reasonableness and limitations of the results.

**Complete a report for the mathematical investigation.**

The report may take a variety of forms, but would usually include the following:

- an outline of the problem and context
- the method required to find a solution, in terms of the mathematical model or strategy used
- the application of the mathematical model or strategy, including
  - relevant data and/or information
  - mathematical calculations and results, using appropriate representations
  - the analysis and interpretation of results, including consideration of the reasonableness and limitations of the results
- the results and conclusions in the context of the problem.

A bibliography and appendices, as appropriate, may be used.

The format of an investigation report may be written or multimodal.

**The report, excluding bibliography and appendices if used, must be a maximum of 15 A4 pages if written, or the equivalent in multimodal form. The maximum page limit is for single-sided A4 pages with minimum font size 10. Page reduction, such as 2 A4 pages reduced to fit on 1 A4 page, is not acceptable. Conclusions, interpretations and/or arguments that are required for the assessment must be presented in the report, and not in an appendix. Appendices are used only to support the report, and do not form part of the assessment decision.**

## Assessment Design Criteria

### Identifies and organises relevant information

<b>A</b>	<p>Identifies and organises information from previous parts of a problem to carry through and solve new and different problems.</p> <p>Defines variables and equations from text and diagrams.</p> <p>Organises data in a concise, clear format and appropriately presents it in tabular, diagrammatic and/or graphical form.</p> <p>Identifies the underlying assumptions related to the relevant mathematics of an investigation.</p>	
<b>B</b>	<p>Identifies and organises key information from previous parts of a problem and brings it together to solve subsequent parts of the problem.</p> <p>Draws a diagram and labels it with appropriate variables.</p> <p>Organises data clearly and appropriately presents it in tabular, diagrammatic and/or graphical form.</p> <p>Identifies suitable variables and constant parameters related to various aspects of an investigation.</p>	
<b>C</b>	<p>Identifies and organises key information needed to solve a familiar problem that is relatively narrow in scope.</p> <p>Identifies variables in a given diagram or draws a simple diagram from given information.</p> <p>Organises some data and presents it in tabular, diagrammatic and/or graphical form.</p> <p>Identifies some mathematical content related to various aspects of an investigation in a given context.</p>	
<b>D</b>	<p>Identifies and organises relevant information</p> <p>Identifies and attempts to use given information to solve a simple routine problem.</p> <p>Identifies variables in a simple diagram.</p> <p>Displays data using an inappropriate presentation format.</p> <p>Identifies limited mathematical content of an investigation.</p>	
<b>E</b>	<p>Does not meet the requirements of a D grade and/or has completed insufficient assessment tasks to be assigned a higher grade.</p>	

## Chooses effective models and methods and carries through the methods correctly

<b>A</b>	Selects an appropriate strategy and applies mathematical knowledge to solve non-routine problems. Generalises and extends models from previous parts of the question. Translates between representations in unpractised ways. Selects appropriate calculator techniques to solve multi-step problems in unfamiliar contexts. Solves unstructured problems and carries through an extended response, using deductive reasoning. Produces results, carries out analysis and generalises in situations requiring investigative techniques.
<b>B</b>	Selects an appropriate strategy and applies mathematical knowledge to solve simple non-routine problems. Translates between representations in practised ways. Selects appropriate calculator techniques to solve multi-step problems. Attempts to analyse and calculate specific cases of generalisation in situations requiring investigative techniques.
<b>C</b>	Selects from a range of strategies and formulae and applies mathematical knowledge in practised ways to solve routine problems. Recognises and uses information in different representations. Uses familiar calculator applications to solve routine problems. Selects an appropriate strategy to carry out analysis in situations requiring investigative techniques.
<b>D</b>	Follows an appropriate strategy to solve practised problems. Deals with information in familiar representations only. Uses a calculator for straightforward problems. Attempts to select an appropriate strategy to carry out analysis in situations requiring investigative techniques.
<b>E</b>	Does not meet the requirements of a D grade and/or has completed insufficient assessment tasks to be assigned a higher grade.

## Follows mathematical conventions and attends to accuracy

<b>A</b>	Consistently uses mathematical conventions to link expressions in clearly defined steps that are easily followed. Works fluently with exact values such as surds, radian values or natural logarithms and exponentials, and expresses answers accurately using these forms as appropriate. Completes concise and accurate solutions to mathematical problems set in applied and theoretical contexts.
<b>B</b>	Interprets and uses mathematical terminology, symbols and conventions in unpractised situations. Defines introduced variables. Uses exact values, such as surds and radian values to express answers accurately when specified. Completes mostly accurate solutions to mathematical problems set in applied and theoretical contexts.
<b>C</b>	Recognises and applies mathematical definitions, rules and procedures in practised situations. Applies basic conventions for diagrams and graphs. Uses calculus, vector and complex number notation correctly. Rounds to suit context and specified accuracy. Generates some accurate and generally complete solutions to mathematical problems set in applied and theoretical contexts.
<b>D</b>	Applies limited mathematical conventions to practised problems. Expresses answers with limited accuracy. Generates partly accurate and generally incomplete solutions to mathematical problems set in applied and theoretical contexts.
<b>E</b>	Does not meet the requirements of a D grade and/or has completed insufficient assessment tasks to be assigned a higher grade.

## Links mathematical results to data and contexts to reach reasonable conclusions

<b>A</b>	Identifies and explains the limitations of complex models. Recognises the domain as implied by the context of the question, and excludes any results outside it. Interprets the result and draws the correct conclusion about the effect of changing conditions. Uses counter-examples and general cases in mathematical analysis of an investigation.
<b>B</b>	Identifies and explains the limitations of simple models. Takes account of the domain as defined in the problem, and excludes results outside it. Links the effect of changing conditions to the original solution. Uses examples in mathematical analysis of an investigation and draws valid conclusions related to a given context.
<b>C</b>	Identifies and describes limitations of simple models. Shows some recognition of the domain as defined in the problem. Recognises that changing conditions will affect the outcome. Makes inferences from analysis and uses these to draw conclusions related to a given context for investigation.
<b>D</b>	Identifies limitations of simple models and, on occasions, recognises specified conditions. Makes some inferences from analysis of an investigation.
<b>E</b>	Does not meet the requirements of a D grade and/or has completed insufficient assessment tasks to be assigned a higher grade.

## Communicates mathematical reasoning, results and conclusions

<b>A</b>	<p>Sets out the steps of the solution using deductive reasoning in a clear and logical sequence, including suitable justification and explanation of methods and processes used.</p> <p>Adds a detailed diagram to illustrate and use in the solution of a problem.</p> <p>Presents work with the final answer clearly identified, using the correct units and related to the context of the question.</p> <p>Communicates investigation findings with a comprehensive interpretation of mathematical results in the context of the investigation.</p>
<b>B</b>	<p>Carries through calculations and simplifications in a clear sequence, showing a logical line of reasoning.</p> <p>Defines variables associated with a given diagram and uses these in the working of a problem.</p> <p>Presents work with the final answer clearly identified and using the correct units.</p> <p>Communicates investigation findings in a systematic and concise way using mathematical language and relating the solution to the original problem or statement.</p>
<b>C</b>	<p>Shows adequate working and supports answers with simple or routine statements.</p> <p>Relates the working to a labelled diagram that has been given as part of the question.</p> <p>Presents work that includes the required answer but does not clearly identify it as such.</p> <p>Communicates investigation findings in a systematic way using some mathematical expression and everyday language.</p>
<b>D</b>	<p>Shows some working in an attempt to answer simple questions.</p> <p>Sets out calculations in a manner that is difficult to check for accuracy.</p> <p>Presents working with no clear indication of the final answer evident.</p> <p>Offers simple conclusions that are not supported by data or calculations.</p>
<b>E</b>	<p>Does not meet the requirements of a D grade and/or has completed insufficient assessment tasks to be assigned a higher grade.</p>